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Possibility of a gravitational effect in the spectra of quasi-stellar objects III

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Abstract. In this concluding paper it is shown that quasi-stellar objects are most probably situated at cosmological distances. The large redshifts of these objects cannot be due to surface gravitational effects. The Hoyle–Fowler model perhaps holds a last hope of attributing redshift to gravity. Different cluster solutions have been analysed to formulate a suitable Hoyle–Fowler model. A modified Hoyle–Fowler model may explain the emission spectrum of quasars, but such a model developed in this paper fails to give a satisfactory explanation of absorption spectra. However, a Hoyle–Fowler model at cosmological distance may possibly explain sharpness and depth of absorption lines. The Hoyle–Fowler model which was originally designed to provide a non-cosmological model of quasi-stellar objects provides a more satisfactory result if it is at a cosmological distance.

1. Introduction

Quasi-stellar objects (QSO) show very large redshifts of spectral lines. The most accepted explanation is that QSO are at cosmological distances. The problems arising out of large energies associated with these objects, the periodic variations of optical and radio flux, the association of QSO with nearby clusters, the anomaly in the redshift–apparent magnitude relation etc, have given rise to the idea that QSO might have some intrinsic redshift. The local hypothesis (Terrel 1964, Hoyle and Burbidge 1966) seems to be highly improbable because of the absence of blueshifted QSO (Faulkner *et al* 1966) and involvement of very high energies (Setti and Woltjer 1966, Bahcall *et al* 1966).

One of the methods by which redshift may possibly be explained can be developed by assuming a massive spherical configuration with large gravitational potential. An analysis by Bondi (1964) leads to the conclusion that if appropriate conditions on stability and equation of state are imposed, z_s (the redshift from the surface) cannot exceed 0·62. The maximum observed redshift is about 3·5 (Wampler *et al* 1973). Even without considering the stability of the configuration, Greenstein and Schmidt (1964) analysed the spectra of QSO 3C273 and 3C48. They concluded from the intensity of forbidden lines and the estimated electron density that QSO having gravitational redshifts can neither exist near the Galaxy nor in the nearby Galactic sphere. But Burbidge (1967) has pointed out in connection with absorption spectra of some QSO that possibly their redshifts are gravitational. These QSO do not exhibit forbidden lines in their emission spectra; hence the analysis given by Greenstein and Schmidt cannot be applied.

Mostly in QSO showing absorption lines, the emission spectra do not show forbidden lines and redshifts of spectral lines are different in absorption and emission. One is then tempted to attribute redshift to a gravitational origin. Recently Morton and Morton (1972) took two spectra of Ton1530 at an interval of one year. They observed that the redshift of absorption lines remains unaltered to an accuracy of 4 in 10^5. If the emission lines are supposed to have a purely surface gravitational redshift then it can be shown by analysing the radial and transverse motion of the absorption clouds (Durgapal 1974b) that the QSO must be at a distance of more than 10^3 Mpc and must have a mass of the order of $10^{17}\,M_\odot$, a highly objectionable result.

However, a Hoyle–Fowler type of model (Hoyle and Fowler 1967) may possibly account for intrinsic redshift. The limitation of the Hoyle–Fowler model is discussed and different cluster solutions have been analysed to develop a suitable model for quasars. It has been shown that even a modified version of the Hoyle–Fowler model does not lead to satisfactory results and certain phenomena cannot be explained. The most suitable conclusion one can derive is that QSO are at cosmological distances.

2. The Hoyle–Fowler (HF) model

In a Hoyle–Fowler type of model the observed emission lines come from the centre of a spherical configuration and $z_{\mathrm{em}} = z_{\mathrm{c}}$. The main mass (in the form of clusters of neutron stars or other highly collapsed objects) serves to generate a strong gravitational field. Further, Hoyle and Fowler suggested that static equilibrium positions are possible in principle for clouds of ions that surround the central cloud. This suggestion would require $z_{\mathrm{em}} > z_{\mathrm{ab}}$ for these static clouds.

2.1. *Isotropic clusters*

Hoyle and Fowler (1967) used Schwarzschild internal solutions with constant density to show that

$$(1+z_{\mathrm{c}})/(1+z_{\mathrm{s}}) = 2/(2-z_{\mathrm{s}}) \tag{1}$$

and that as $P_{\mathrm{c}} \to \infty$, $z_{\mathrm{s}} \to 2$ and $z_{\mathrm{c}} \to \infty$. Fackrell (1968) has proved that an isotropic cluster with constant density cannot be stable since the distribution function becomes negative for

$$\tfrac{1}{2}(3-\sqrt{3})^{1/2}\, \mathrm{e}^{\frac{1}{2}\nu(a)} mc^2 < E \leqslant \mathrm{e}^{\frac{1}{2}\nu(a)} mc^2 \tag{2}$$

where e^{ν} has its usual meaning (that is, the factor which appears in the metric

$$\mathrm{d}s^2 = \mathrm{e}^{\nu(r)}\,\mathrm{d}t^2 - \mathrm{e}^{\lambda(r)}\,\mathrm{d}r^2 - r^2\,\mathrm{d}\theta^2 - r^2 \sin^2\theta\,\mathrm{d}\phi^2).$$

Ipser (1969) showed that relativistic polytropic clusters with polytropic indices 2 and 3 are unstable if the redshift from the centre (z_{c}) exceeds 0·5. Fackrell (1970) puts the limit for stability at $z_{\mathrm{c}} \leqslant 0{\cdot}7302$ by considering spheres with polytropic index 4. Fackrell arrived at this limit of z_{c} by considering an extreme core–halo density distribution and it would not be improper to consider this as an upper limit for stable isotropic clusters. Though Tolman's (1939) IV, V and VI solutions indicate the possibility of infinitely large z_{c}, their stability is doubtful. Solution IV has a negative distribution function (this can be seen by using Fackrell's method of 1968).

By considering a spherical configuration of infinite radius, Tolman's V and VI solutions reduce to self-similar solutions (Kogan and Zeldovich 1969) with

$$\begin{aligned} P &= \gamma\rho = 4\gamma^2/8\pi r^2(\gamma^2+6\gamma+1), \\ e^\nu &= \text{constant} \times r^{4\gamma/(\gamma+1)}, \qquad e^\lambda = \text{constant}. \end{aligned} \tag{3}$$

Kogan and Thorne (1970) showed that the most powerful techniques yet devised yield inconclusive results for stability of these (self-similar) clusters. The difficulty with this model is that one always gets infinite central redshift (from equation (3)). However, one may construct a two-density distribution to avoid this. But then the model will become too complicated to explain various properties of quasars; moreover, the infinitely large radius will make the model unsuitable.

2.2. *Circular orbits*

For circular orbits, $T'_{rr} = T_{rr} = 0$ so that $\nu' = (e^\lambda - 1)/r$ (Zeldovich and Novikov 1971) where a prime stands for d/dr. Recently Florides (1974) has discussed the physical rationality of such solutions. With a given $\rho(r)$ the functions $\nu(r)$ and $\lambda(r)$ may be readily determined. Without going into the details of calculations the metrics for different density distributions are given below. For the sake of brevity we may put

$$k = m/a, \qquad x = r^2/a^2 \qquad \text{and} \qquad d\Omega^2 = r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{4}$$

where m and a are the mass and radius of the spherical configuration.

(i) Density $\propto r^{N-2}$ (where $N > 0$). The metric is given by

$$ds^2 = \frac{(1-2k)^{1+1/N}\, dt^2}{[1-2k(r/a)^N]^{1/N}} - \frac{dr^2}{1-2k(r/a)^N} - d\Omega^2. \tag{5a}$$

For $N = 2$ the density is constant and we get

$$ds^2 = (1-2k)^{3/2}(1-2kx)^{-1/2}\, dt^2 - (1-2kx)^{-1}\, dr^2 - d\Omega^2. \tag{5b}$$

This metric is identical to the solution obtained by Florides (1974) for a constant density sphere with zero radial stress.

(ii) $e^\nu \nu'/r^{(1/n)-1}$ = constant (where e^ν has its usual meaning and $\nu' = d\nu/dr$). One can easily obtain (see Tolman's IV solution)

$$ds^2 = [1-2(n+1)k + 2nk(r/a)^{1/n}]\, dt^2 - \frac{1-2(n+1)k+2(n+1)k(r/a)^{1/n}}{1-2(n+1)k+2nk(r/a)^{1/n}}\, dr^2 - d\Omega^2 \tag{6}$$

and density

$$\rho = \frac{2(n+1)k(r/a)^{1/n}[1-2(n+1)k+2k(r/a)^{1/n}]}{8\pi n r^2[1-2(n+1)k+2(n+1)k(r/a)^{1/n}]^2}. \tag{7}$$

For values of $n > \frac{1}{2}$ the density at the centre will be infinite, while for $n < \frac{1}{2}$ the density increases as one moves out from the centre to the surface. Only when $n = \frac{1}{2}$, the density is positive finite throughout the configuration and decreases from centre to surface.

The metric for $n = \frac{1}{2}$ is

$$ds^2 = (1-3k+kx)\,dt^2 - (1-3k+3kx)(1-3k+kx)^{-1}\,dr^2 - d\Omega^2 \tag{8}$$

$$\rho = \frac{3k(1-3k+2kx)}{4\pi a^2(1-3k+3kx)^2} \quad \text{and} \quad \frac{\rho(\text{surface})}{\rho(\text{central})} = (1-k)(1-3k). \tag{9}$$

The metric for $n = 2$ is

$$ds^2 = [1-6k+4k\sqrt{(r/a)}]\,dt^2 - \frac{1-6k+6k\sqrt{(r/a)}}{1-6k+4k\sqrt{(r/a)}}\,dr^2 - d\Omega^2. \tag{10}$$

This metric is important when one considers stable orbits.

(iii) Density $\propto r^{-2}$ (similar to Tolman's V and VI solutions and the self-similar solution of equation (3)). This solution has been discussed by Zeldovich and Novikov (1971) to show the possibility of infinitely large redshift. The metric can be written as

$$ds^2 = (1-2k)x^n\,dt^2 - (1-2k)^{-1}\,dr^2 - d\Omega^2 \tag{11}$$

where

$$n = k/(1-2k). \tag{12}$$

(iv) Density $\propto 1 - r^2/a^2$ (see Tolman's VII solution). Using equation $\nu' = (e^\lambda - 1)/r$ and the results of Durgapal and Gehlot (1971), we obtain

$$ds^2 = e^\nu\,dt^2 - \frac{dr^2}{1-kx(5-3x)} - d\Omega^2 \tag{13}$$

where

$$\nu = \frac{1}{4}\ln\left(\frac{(1-2k)^5}{1-kx(5-3x)}\right) - \frac{5}{2}\left(\frac{k}{12-25k}\right)^{1/2}\tan^{-1}\left(\frac{\sqrt{[k(12-25k)](1-x)}}{2-5k+kx}\right). \tag{14}$$

2.3. *Limitation on clusters with circular orbits*

It is well known that no material particle can move in a circular orbit in a Schwarzschild exterior field of radius $a \leqslant 3m$. Thus for solutions with $T'_{rr} = T_{rr} = 0$ we put the restriction (Florides 1974)

$$a > 3m. \tag{15}$$

A more reasonable limit (Zapolsky 1968) can be arrived at by assuming that different stars in the cluster are moving in stable orbits. For a stable circular orbit,

$$r \geqslant 6m(r) \tag{16}$$

where $m(r)$ is the mass contained within the radius r. It can be seen from § 2.2 that condition (16) reduces to

$$a \geqslant 6m. \tag{17}$$

An analysis of different solutions has been done and the results are summarized in table 1.

Table 1. Central redshift for different density distributions.

	Central redshift when $a = 3m$	$a = 6m$
Density = constant	1·279	0·355
Density $\propto r^{-2}$	∞	∞
Density $\propto 1-r^2/a^2$	1·995	0·483
$e^{\nu}\nu'/r^{(1/n)-1}$ = constant $n=\frac{1}{2}$	∞	0·414
$e^{\nu}\nu'/r^{(1/n)-1}$ = constant $n=2$	—	∞

2.4. *Model for quasars*

From table 1 it is clear that the cluster structures which can provide us with a proper local HF model are: (i) solutions with density $\propto$ (radius)$^{-2}$, and (ii) solutions with $e^{\nu}\nu'/r^{(1/n)-1}$ = constant.

(i) $\rho \propto 1/r^2$ (see also self-similar solutions, equation (3)). The difficulty with this solution is that the central density is infinity and one always gets infinite central redshift. The redshift of quasars is finite. One can assume a suitable radius $r = \Delta a$ for an emission cloud such that

$$1+z_{\mathrm{em}} = (2n+1)^{1/2}(a/\Delta a)^n \tag{18}$$

where z_{em} is the emission redshift of the quasar. In the HF model $\Delta a \simeq 10^{-2}a$, thus

$$10^{2n}(2n+1)^{1/2} = 1+z_{\mathrm{em}}. \tag{19}$$

The lines originating from the interior of the emission cloud will be more redshifted than those originating at $r = \Delta a$, so much so that the lines originating at $r = 0$ are infinitely redshifted (at $N_e = 10^4$–10^6 and $\Delta a \simeq 10^{-2}$ pc the forbidden lines have very low optical depth). The extremely large width of emission lines will make the spectrum featureless. Thus this solution is unable to provide us with a suitable model of QSO.

(ii) $e^{\nu}\nu'/r^{(1/n)-1}$ = constant. For $n = \frac{1}{2}$ this solution corresponds to a cluster with positive finite density throughout (equation (9)). It is possible to construct a model of a quasar with any large redshift. The central redshift is given by (from equation (8))

$$1+z_c = \left(1-\frac{3m}{a}\right)^{-1/2} \tag{20}$$

and the redshift z_r at a distance r from the centre is

$$1+z_r = \left(1-\frac{3m}{a}+\frac{m}{a}\frac{r^2}{a^2}\right)^{-1/2}. \tag{21}$$

From equations (20) and (21) it can be shown that the width w of emission lines due to change in gravitational potential across the central emission cloud is given by

$$\frac{w}{\lambda} = 2\frac{m}{a}\left(\frac{\Delta a}{a}\right)^2\left(1-\frac{3m}{a}\right)^{-1}. \tag{22}$$

If there is a static absorption cloud at a distance r from the centre, it can easily be shown that

$$r/a = [3(f^2-1)/z_{\mathrm{em}}(2+z_{\mathrm{em}})]^{1/2} \tag{23}$$

where

$$f = (1+z_{em})/(1+z_{ab}). \tag{24}$$

This model (equations (20) to (23)) has been applied in § 3.2 to discuss the absorption spectra of some quasars.

If we consider the stability condition as $a \geqslant 6m$, the solution with $n = 2$ should be used. The drawback with this solution is the existence of infinite central density. However, the mass contained in a finite volume is finite. The central redshift is given by

$$1+z_c = \left(1-\frac{6m}{a}\right)^{-1/2} \tag{25}$$

and

$$\frac{w}{\lambda} = \frac{2m}{a}\left(\frac{\Delta a}{a}\right)^{1/2}\left(1-\frac{6m}{a}\right)^{-1}. \tag{26}$$

As compared to equation (22) we get about 1000 times broader lines from equation (26). Hence this model is not as suitable as that with $n = \frac{1}{2}$ in discussing the spectra of quasars.

3. Absorption spectra and HF models

One of the most important features of the absorption spectra is the sharpness of absorption lines. A model of QSO must be such that the gravitational linewidth does not exceed $30\,\mathrm{km\,s^{-1}}$ ($w/\lambda \lesssim 10^{-4}$) or so.

3.1. Width of absorption lines in HF models

Let the cosmological redshift of a QSO be Z; then

$$\lambda/\lambda_0 = (1+Z)\,e^{-\frac{1}{2}\nu(r)} \tag{27}$$

where

λ = observed wavelength of absorption line
λ_0 = unshifted wavelength of absorption line
r = distance of absorption cloud from the centre.

In different models the linewidth will be different.

(i) Constant density HF model

$$e^{\nu(r)} = \left(1-\frac{2m}{a}\right)^{3/2}\left(1-\frac{2m}{a}\frac{r^2}{a^2}\right)^{1/2}. \tag{28}$$

Now,

$$\lambda_0/\lambda' - \lambda_0/\lambda = (1+Z)^{-1}\left(e^{\frac{1}{2}\nu(r')} - e^{\frac{1}{2}\nu(r)}\right) \tag{29}$$

or

$$\frac{w}{\lambda} = \frac{r}{a}\frac{\Delta r}{a}\frac{m}{a}\left[1-\frac{2m}{a}\left(\frac{r}{a}\right)^2\right]^{-1} \tag{30}$$

where

$$w = \lambda - \lambda' \ll \lambda \quad \text{and}$$

$$r' - r = \text{thickness of absorption cloud} = \Delta r \ll r.$$

(ii) Varying density HF model with $e^{\nu} = \text{constant} \times r^{2n}$ (Durgapal 1974c, Zeldovich and Novikov 1971 and equations (3), (11) and (12) of this paper). From

$$e^{\frac{1}{2}\nu(r)} = (2n+1)^{-1/2}(r/a)^n \tag{31}$$

we have

$$\begin{aligned} w/\lambda &= (1+z_g)\, e^{\frac{1}{2}\nu(r)}(e^{\frac{1}{2}\nu(r')}/e^{\frac{1}{2}\nu(r)} - 1) \\ &= (r'/r)^n - 1 = n\Delta r/r \qquad (32) \\ &= (m/a)(\Delta r/r)(1-2m/a)^{-1} \qquad (33) \end{aligned}$$

where

$$m/a = n/(2n+1). \tag{34}$$

(iii) Modified HF model with $e^{\nu}\nu'/r$ = constant (§ 2.4(ii)). For this model (from equation (8))

$$e^{\nu(r)} = 1 - \frac{3m}{a} + \frac{m}{a}\left(\frac{r}{a}\right)^2. \tag{35}$$

Now,

$$\frac{\lambda_0^2}{\lambda'^2} - \frac{\lambda_0^2}{\lambda^2} = (1+Z)^{-2}(e^{\nu(r')} - e^{\nu(r)})$$

or

$$\frac{w}{\lambda} = (1+z_g)^2 \frac{m}{a}\frac{r}{a}\frac{\Delta r}{a}. \tag{36}$$

The gravitational width depends upon the ratios m/a, r/a and $\Delta r/a$.

3.2. HF model at small distance

The absorption spectra of QSO Ton1530 and 4C05.34 have been discussed by using the modified model of § 3.1(iii) (equations (20)–(23), (35), (36)).

(i) Ton1530. $z_{em} = 2{\cdot}047$. Distances of the two absorption clouds are $r/a = 0{\cdot}13$ (for $z_{ab} = 1{\cdot}9803$) and $0{\cdot}156$ (for $z_{ab} = 1{\cdot}93702$). A peculiarity is the observation of a triple feature ($z^+ = 1{\cdot}9358$, $z^0 = 1{\cdot}9371$, $z^- = 1{\cdot}9384$) in the C IV absorption lines. The splitting $\Delta z = \pm 0{\cdot}0013$ on either side of the central line corresponds to a distance $d = \pm 0{\cdot}001a$ of satellite clouds from the z^0 cloud.

The width of absorption lines due to variation of gravitational potential across the cloud is given approximately from equation (36) by $0{\cdot}4\Delta r/a \leqslant 10^{-4}$ (for $\Delta v_D = 40$ km s^{-1}). Hence

$$\Delta r \leqslant 2{\cdot}5 \times 10^{-4}a \quad \text{and} \quad \Delta r/d \leqslant \tfrac{1}{4}. \tag{37}$$

The column density of the C IV line is of the order of 10^{14} cm^{-2}. If $a = 1$ pc, the number of ions per cm^3 is of the order of at least 10^{-1}. Absence of any transition from the excited level of the fine structure ground state is consistent with this particle density.

The model fails to give an explanation of the fact that the residual intensity in some cases is almost zero, because

$$\frac{\text{Size of absorption cloud}}{\text{Size of central cloud}} = \frac{\Delta r}{\Delta a} \times 10^{-2}.$$

It is unfair to assume a plate-like cloud with radius Δa and thickness Δr. This is a serious drawback of a local HF object.

(ii) 4C05.34. The futility of the HF model is shown in explaining the spectrum of 4C05.34 which has $z_{em} = 2{\cdot}877$ and $z_{ab} = 2{\cdot}8751, 2{\cdot}8106, 2{\cdot}7703, 2{\cdot}5925, 2{\cdot}4743, 2{\cdot}1819, 1{\cdot}8593, 1{\cdot}7758$. These absorption features occur at $r/a = 0{\cdot}014, 0{\cdot}09, 0{\cdot}11, 0{\cdot}19, 0{\cdot}23, 0{\cdot}32, 0{\cdot}43$ and $0{\cdot}45$ respectively. Transitions are from fine structure ground states of C II, N II, Si II, Fe II and Fe III. No lines from excited fine structure are definitely identified. Hence one can say (Bahcall and Goldsmith 1971) that $N_e \lesssim 10^2$ and $D \gtrsim \epsilon \times 10^3$ pc, where D is the distance of the absorption cloud from the QSO continuum and

$$\epsilon = \frac{\text{Actual distance of QSO}(R)}{\text{Distance of QSO if redshift is due to cosmological expansion}}$$

Hence

$$10^{-2}a \gtrsim \epsilon \times 10^3 \text{ pc} \qquad \text{or} \qquad a \gtrsim \epsilon \times 10^5 \text{ pc}.$$

If $R = 1$ to 10 Mpc, $\epsilon \simeq 10^{-3}$ giving $a \simeq 10^2$ pc and $m \simeq 10^{15} M_\odot$. A huge mass of this magnitude at such a short distance is highly improbable. If $R = 100$ Mpc, $\epsilon \gtrsim 10^{-2}$ and $a \gtrsim 10^3$ pc with a mass $m \gtrsim 10^{16} M_\odot$. Less than 10^3 such objects will be sufficient to account for the entire *missing mass* (Sandage 1965 estimated the number of QSO at about 10^5).

Similar difficulties will arise in explaining the absorption spectra of PHL957 and PKS0237-23. Thus in absorption spectra of QSO it is difficult to explain (i) the depth and sharpness of absorption lines simultaneously and (ii) the absence of transitions from excited levels of fine structure ground states, if they are local HF objects. If the HF models with $\rho \propto 1/r^2$ or $\rho \propto r^{N-2}$ (with $N > 0$) are applied to the above cases, the results will be even more disappointing.

3.3. *HF model at cosmological distance*

The author (Durgapal 1974a) has already discussed such a possibility. In this section an attempt has been made to explain the sharpness of absorption lines. Since the residual intensity of many absorption lines reaches zero, the size of the absorption cloud $\Delta r \simeq \Delta a$. Since $\Delta a/a \simeq 10^{-2}$, $r/a \leqslant 1$ and $m/a = 0{\cdot}01$ to $0{\cdot}02$ in most of the cases. Equations (30) and (36) give $w/\lambda \simeq 10^{-4}$ in most of the cases. Thus both the sharpness and depth of absorption lines may possibly be explained in many (not all) QSO by considering the HF model at cosmological distances.

However, the phenomenon $z_{ab} > z_{em}$ has to be explained by considering absorption clouds falling towards the QSO. The analysis of an earlier paper (Durgapal 1974b) can be applied to the modified HF model with $e^{\nu}\nu'/r =$ constant if the expression for gravitational redshift (Durgapal 1974b, equations (8) and (16)) z'_{em} is replaced by

$$1 + z'_{em} = (1 - 3m/a)^{-1/2}. \tag{38}$$

4. Conclusions

From the experiment of Morton and Morton (1972) and conclusions of Greenstein and Schmidt (1964) we may say that, irrespective of the presence of forbidden lines, QSO with purely gravitational surface redshift can neither exist near the Galaxy nor in the nearby Galactic sphere. The Hoyle and Fowler (1967) model was proposed to avoid objections raised by Greenstein and Schmidt. An analysis of the HF model has been done by considering different types of solutions. As far as the emission spectrum is concerned it is possible to explain large redshifts of quasars provided the condition for stability is taken as $a > 3m$. As for the absorption spectrum, the solutions discussed in this paper are unable to provide a local HF model which can simultaneously explain the sharpness and depth of the absorption lines. However, the HF model at cosmological distance may satisfactorily explain the sharpness and depth of absorption lines in many (not all) QSO. Thus we see that even HF models provide a better explanation of absorption spectra when they are at cosmological distances.

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